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ANALOG SIMULATION OF METAL HEATING WITH SCALE FORMATION

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The heat conduction problem for a metal heated in a furnace with scale formation is solved by analog simulation on a resistance network. The reciprocal effect of oxidation of the metal on the heating process and of the temperature regime on the kinetics of scale formation is taken into account.

The heating of metals in high-temperature furnaces is accompanied by oxidation of the surface and the formation of scale. The heating and scale-forming processes are a result of heat and mass transfer between the metal and the gas medium in the furnace and are interdependent. The rate of scale formation depends on the temperature regime of the metal surface [1] and other factors. There is a feedback: the heat of oxidation released at the surface of the body and the thermal resistance of the layer of scale, which has a thermal conductivity smaller by a factor of 20–30 than that of the metal, affect the heating regime. In engineering practice, the heating and oxidation processes are considered separately. Thus, analytic methods of calculating metal heating in furnaces [2] do not take into account the effect of scale formation on heat transfer and are wholly suitable only for the case of nonoxidative heating. Below, we present a method of solving the combined problem of heating and oxidation of a cylindrical ingot of infinite length with boundary conditions of the third kind (Fig. 1). A layer of scale grows at the surface of the ingot with the release of heat of oxidation at the scale-furnace interface.

The rate of scale formation is determined by diffusion processes and is expressed by the following equation [1] derived from Fick's law:

$$\frac{dM}{d\tau} = \frac{\frac{k_0}{2}\exp(-B/\overline{T}_2)}{M}. \tag{1}$$

The thickness of the scale is related to the mass of metal converted into scale by the following expression:

$$\delta = \frac{1.33M}{\gamma_2}. \tag{2}$$

The power of the source of heat of oxidation

$$q_{\text{ox}} = Q_{\text{ox}}\frac{dM}{d\tau}. \tag{3}$$

The solution must give the temperature field of the ingot and the growth kinetics of the layer of scale during heating $\delta = f(\tau)$.

In this case, the determination of the temperature field of the metal is a problem of heat conduction of a two-layer scale-metal body with variable thickness of the layers. The effect of oxidation on the ingot heating process is expressed as the variation in time of the thickness of the scale and metal layers and by introducing into the boundary conditions the source of heat of oxidation.

The system of differential equations of heat conduction has the following form: heat conduction equation of the metal

$$\frac{\partial t_1}{\partial \tau} = a_1\left(\frac{\partial^2 t_1}{\partial r^2} + \frac{1}{r}\frac{\partial t_1}{\partial r}\right) \tag{4}$$
$$(\tau > 0;\ 0 \leqslant r \leqslant S);$$

heat conduction equation of the scale

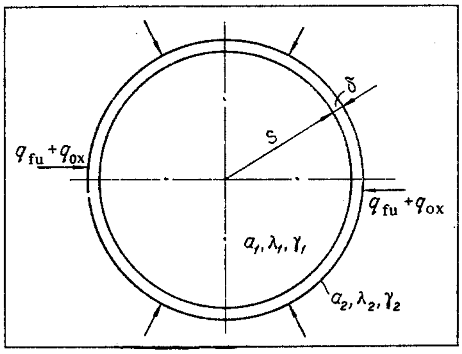

Fig. 1. Heating of an ingot with scale formation.

$$\frac{\partial t_2}{\partial \tau} = a_2 \frac{\partial^2 t_2}{\partial r^2} \quad (\tau > 0;\ S \leqslant r \leqslant S + \delta). \tag{5}$$

The layer of scale is assumed to be plane because of the small ratio of thickness to radius.

The boundary conditions are

$$\alpha\,[t_{\mathrm{fu}} - t_2(S + \delta,\ \tau)] + Q_{\mathrm{ox}} \frac{dM}{d\tau} = \lambda_2 \frac{\partial t_2 (S + \delta,\ \tau)}{\partial r}, \tag{6}$$

$$t_1(S,\ \tau) = t_2(S,\ \tau); \quad \lambda_1 \frac{\partial t_1(S,\ \tau)}{\partial r} = \lambda_2 \frac{\partial t_2(S,\ \tau)}{\partial r}. \tag{7}$$

The initial conditions are

$$t_1(r,\ 0) = f(r);\ t_2(r,\ 0) = t_1(S,\ 0).$$

At time $\tau = 0$ there is an initial film of scale $\delta(0) \neq 0$ on the surface of the metal, so that from the outset the process of scale formation satisfies the differential equation (1).

The heat conduction problem was solved by analog simulation on a resistance network [3, 4]. This network and the corresponding finite-difference scheme are shown in Fig. 2. The layer of scale has been treated as a single elementary layer with a node at the center of the thickness. The network resistances were determined by the method described in [4].

There was no need to simulate the oxidation kinetics. This problem was solved by calculation using values of the scale temperature measured in the process of simulating the heating of the ingot. The quantities characterizing scale formation were computed after each measurement of the electric potentials, and the network resistances were modified according to the results of the calculation before each new measurement.

The following order of operations at the i-th moment of time was adopted:

1) mass of metal converted into scale:

$$M_i = M_{i-1} + \left(\frac{dM}{d\tau}\right)_{i-1} \Delta\tau; \tag{8}$$

2) decrease in the thickness of the surface layer of metal due to the diffusion of ions into the oxide phase:

$$\nu_i = \nu_0 - \frac{M_i}{\gamma_1}; \tag{9}$$

3) thickness of the layer of scale in accordance with (2)

$$\delta_i = \frac{1.33\,M_i}{\gamma_2}; \tag{10}$$

4) rate of oxidation of the metal in accordance with (1)

$$\left(\frac{dM}{d\tau}\right)_i = \frac{\frac{k_0}{2} \exp\left(-B/\overline{T}_{2(i)}\right)}{M_i}; \tag{11}$$

5) specific oxidation heat flux from (3)

$$q_{\mathrm{ox}(i)} = Q_{\mathrm{ox}} \left(\frac{dM}{d\tau}\right)_i. \tag{12}$$

The model resistances were corrected in accordance with the values of δ_i, ν_i and $q_{ox(i)}$ obtained.

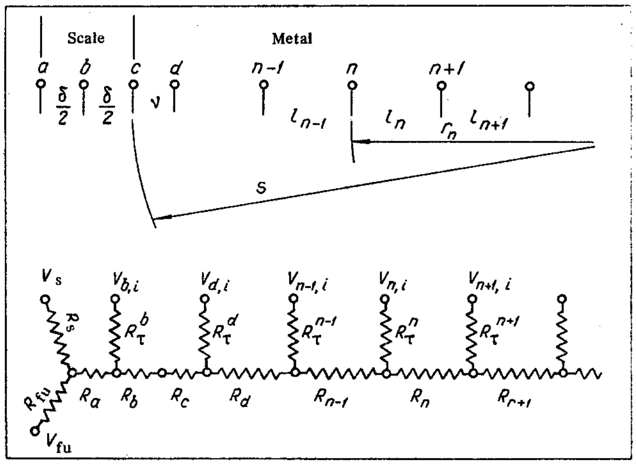

Fig. 2. Finite-difference scheme and resistance network.

Thus, by combining analog simulation with running oxidation calculations we took into account the reciprocal influence of scale formation on the temperature and thermal heating regimes and of the temperature regime on the growth kinetics of the layer of scale.

The temperature (a) and thermal (b) heating regimes, obtained on an EI-12 electrointegrator as a result of simulating heating with allowance for scale formation, are presented in Fig. 3 for a cylindrical ingot of low-carbon steel. The curve in Fig. 3c represents the growth kinetics of the scale layer, obtained by calculation from formulas (8)–(12) using the temperature at the scale node measured during simulation.

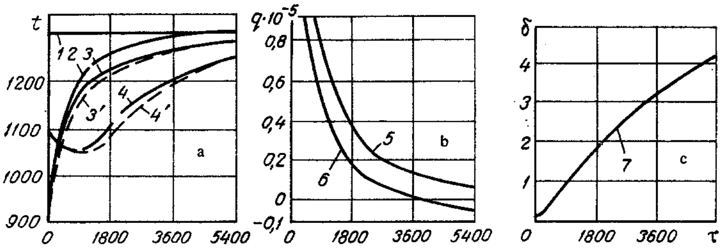

Fig. 3. Heating and oxidation of ingot (t, °C; q, W/m^2; δ, mm; τ, sec): 1) furnace temperature; 2) temperature of outer surface of scale; 3) temperature of ingot surface during heating with oxidation; 3') the same without oxidation; 4) temperature on ingot axis during heating with oxidation; 4') the same without oxidation; 5) total heat flux $q_{fu} + q_{ox}$; 6) heat flux q_{fu}; 7) growth of scale layer.

In the given example, we specified the following values. Ingot diameter 0.6 m. The initial temperature field of the ingot is described by the parabola

$$t_1(r,\ 0) = 1100 - 300\left(\frac{r}{S}\right)^2\ ^\circ\mathrm{C}.$$

The surface temperature of the ingot at the end of heating is 1280° C; the furnace temperature is 1300° C. Heat is transmitted from the furnace to the ingot by radiation. The reduced radiation factor is 4.07 W/m$^2\cdot$deg^4. The density of the steel γ_1 = 7500 kg/m^3; that of the scale γ_2 = 5000 kg/m^3. The dependence of λ_1 (W/m · deg) and c_1 (kJ/kg · deg) on temperature (°C) is as follows:

t	0÷1000	1000÷1200	1200÷1300
λ_1	46.5	44.5	46.5
c_1	0.675	0.78	0.71

The temperature dependence of λ_2 (W/m · deg)[5]:

t	900	1000	1100	1200	1300
λ_2	1.45	1.62	1.85	2.1	2.3

$c_2 = 1.25$ kJ/kg · deg; $\delta(0) = 0.0001$ m.

The values of the coefficients $k_0 = 5 \cdot 10^3$ kg^2/m^4 · sec and $B = 18 \cdot 10^3$ deg were taken from the experimental data. The heat of oxidation of the steel $Q_{ox} = 5700$ kJ/kg.

The model space and time intervals were selected by comparing the results of simulating nonoxidative heating with the known analytic solution. An accuracy sufficient for practical calculations was obtained at $\Delta\tau = 180$ sec and $l = 0.06$ m.

As follows from Fig. 3a, the temperature regime with allowance for oxidation (curves 3 and 4) differs somewhat from the corresponding nonoxidative regime (curves 3' and 4'). At the same time, the oxidation process does not affect the time required to heat the ingot to a given end state. The surface temperature of the scale (curve 2) in the final stage of heating is greater than the furnace temperature owing to the release of heat of oxidation, thanks to which the radiative heat flux q_{fu} (Fig. 3b, curve 6) is negative during this period. A comparison of the total heat flux $q_{fu} + q_{ox}$ (curve 5) and the quantity q_{fu} points to the considerable role of the heat of oxidation in the absorption of heat by the metal. The heat of oxidation corresponds approximately to one-third of the total heat absorbed by the ingot.

NOTATION

t_1 is the temperature of the metal; t_2 is the temperature of scale; $\overline{T}_2$ is the mean-mass temperature of the scale; S is the ingot radius; r is the coordinate; τ is the time; δ is the thickness of the scale layer; a_1, λ_1, γ_1 are the thermal diffusivity, thermal conductivity, and density of the metal; a_2, λ_2, γ_2 are the same for the scale; t_{fu} is the furnace temperature; α is the external heat-transfer coefficient; q_{fu} is the specific heat flux from the furnace to the scale surface; q_{ox} is the specific oxidation heat flux; M is the mass of metal in the scale per m^2 ; k_0, B is a coefficient which depends on the oxidizing capacity of the furnace gases and the properties of the metal; Q_{ox} is the heat of oxidation per kg of metal; ν and l are space intervals ; $\Delta\tau$ is the time interval; R is the electrical resistance between nodes; R_τ is the time resistance ; V is the electric potential.

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